# Efficient parameter estimation techniques for nonlinear MEMS resonators

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***Abstract*— This paper highlights experimental results obtained during the CARAC-ATAC research project. The aim of the project was to investigate techniques for performing fast characterization of high-quality factor, low-frequency MEMS linear or nonlinear resonators, such as energy harvesters, resonant strain gauges, gyroscopes, etc. Here we focus on the use of ring-down and of fast frequency sweeps to estimate the parameters of resonators operating in the nonlinear regime. We show that either approach may generally provide equally-accurate and precise estimates of most of a nonlinear resonator's parameters, with trade-offs in terms of experimental complexity and number of estimated parameters. We also show that nonlinear ring-down fails at correctly estimating nonlinear damping parameters unless prior knowledge of the resonator's frequency is available.**

***Keywords—MEMS, resonator, characterization, nonlinear ring-down, fast frequency sweeps***

## I. Introduction

Experimentally assessing a MEMS resonator's parameters, such as its natural frequency $f_0$ or quality factor $Q$, presents several interesting challenges. For example, parameter estimation is required for testing and calibrating the resonator's response at the production stage; it is also a useful laboratory tool for the fine characterization of resonators, and may even be put to practical use in a variety of applications (e.g. parametric / resonant sensing, tuning of gyroscopes or energy harvesters). Independently of the context, it is preferable to limit the duration of this experimental procedure. For example, in the manufacturing process of MEMS resonators, long testing times, requiring stringent control of environmental conditions over a long duration, result in increased device cost, and may even lead to production bottlenecks. During the system lifetime, when environmental drift is not controlled, it is also essential that re-calibration be performed on the basis of data that has been acquired over a short window of time. A good parameter estimation technique should then be able to rely on transient rather than steady-state data, and yield precise results with moderate computational effort –in view of an embedded application, processing should in fact be kept as simple as possible. Other very desirable features are the possibility of using the same technique regardless of the exact model of the resonator (e.g. nonlinear resonators, coupled resonators), of using it in parallel on a batch of resonators, and of using it with as little prior information as possible.

To obtain quantitative information about a resonator, steady-state parameter estimation techniques rely on the measurement of $N$ points of the – linear or nonlinear – frequency response [1] (see [2-4] for typical uses in a MEMS context). This experimental procedure requires a time that is significantly larger than $N \times \tau$, where $\tau = Q/\pi f_0$ is the characteristic response time of the resonator. In many MEMS applications related to inertial forces, such as inertial sensing [5] or kinetic energy harvesting [6], $\tau$ is typically on the order of 0.1 to 10 seconds. During the experimental procedure, environmental conditions must then be precisely monitored, and controlled or compensated for, otherwise drift would affect the parameter estimation results. To alleviate this issue, closed-loop control techniques can significantly reduce the time required for the resonator to reach steady-state, and consequently determine its frequency response (from several hours to a few minutes in [7]). While this control-based approach may in theory be applied to nonlinear resonators, there exists, to the best of our knowledge, no work in this direction. Furthermore, the extension of this closed-loop technique to the simultaneous test of multiple resonators appears rather challenging.

Alternatively, some methods exploit transient measurements to perform parameter estimation with a reasonable time scale, the golden standard for high $Q$, low $f_0$ resonators being the "ring-down" method, which consists in measuring the decaying oscillations of a free resonator. Several variants of this method have been proposed in recent years, extending its use from the case of a single degree-of-freedom resonator to multiple degrees-of-freedom, and/or nonlinear resonators [8-10]. Ring-down methods are faster than steady-state methods, and they are intrinsically immune to "feedthrough", i.e. parasitic coupling between the driving and sensing terminals of the resonator. Yet, they also have several limitations: some parameters cannot be estimated (e.g. transduction gain and transduction nonlinearity), and their on-chip implementation may require, as in [8-9], the solution of a nonlinear least squares problem, with no guarantee of convergence. Furthermore, the resonator must be launched with a significant initial amount of energy, either by applying to it a large DC stimulus or by starting it from a "high" resonant state, both of which may be, depending on the context, impractical: the former requires the generation of a very large DC force ($Q$ times larger than that required to drive the system at resonance). In particular, simultaneously performing ring-down measurements from an initially resonant state on a batch of resonators with different - and a priori unknown - frequencies is extremely challenging.

We have introduced in [11] an approach for estimating resonator parameters from fast frequency sweeps and illustrated it with simulations. In theory, this approach meets several of the demands of a "good" parameter estimation technique, as it is an open-loop technique with low computational complexity, which yields precise results provided the sweeping time is – at least – of the order of τ, and which may be used for linear and nonlinear resonators alike. It shares the same "Hilbert transform" formalism as the nonlinear ring-down method described in [8-9], or the non-parametric FORCEVIB and FREEVIB methods [12-13] initiated by Feldman in the field of nonlinear vibration analysis.

In this paper, we illustrate its experimental use for the first time, in the case of a resonant pressure sensor operated in the nonlinear regime. We show that frequency sweeps generally provide accurate and precise estimates of most of a nonlinear resonator's parameters, with trade-offs in terms of experimental complexity and number of estimated parameters compared to nonlinear ring-down. We also show that nonlinear ring-down fails at correctly estimating nonlinear damping parameters unless prior knowledge of the resonator's frequency is available, whereas frequency sweeps provide robust estimates.

The paper is structured as follows. Section II describes the studied device and its model. Section III focusses on the experimental setup and the data processing. The results obtained with ring-downs and sweeps are presented in section IV. Section V is dedicated to concluding remarks and perspectives.

## II. Device under test

### A. Decription of the device

Thales' P90 pressure sensor [14] is based on a silicon resonant strain gauge fixed to a pressure-sensing diaphragm at one end, and clamped at the other (Fig. 1). Pressure acting on the diaphragm results in axial stress in the resonator beam, which modulates its resonance frequency (close to 70kHz at ambient pressure). The resonator is enclosed in a microfabricated vacuum chamber; its nominal quality factor is on the order of 20000. Thus, the nominal characteristic time $\tau$ is 91ms and the nominal bandwidth $\Delta f$ is 1.75Hz. Capacitive actuation and detection are used to set the beam in an oscillating state and sense its motion.

This resonator exhibits strong nonlinearity when subject to large DC bias or AC drive [15]. Furthermore, because of its geometry, a large amount of capacitive feedthrough is unavoidable.

### B. Model of the device

As a first approximation, the device can be modeled as a one-port resonator with mass $m$, equivalent stiffness $k$, viscous damping $b$, cubic restoring and damping forces, and a driving force proportional to the drive voltage $v_{drv}$:

$$\ddot{x} \approx -\omega_0^2(1+\gamma x^2)x - \frac{\omega_0}{Q}(1+\alpha x^2)\dot{x} + g_{drv}v_{drv} \quad (1)$$

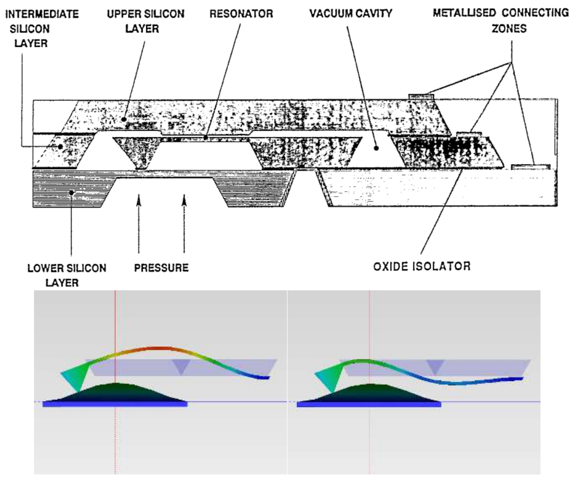


Fig. 1. Cross-sectional view of the P90 presure sensor (top, from [14]) and modal analysis simulation (bottom) showing the static shape taken by the membrane at ambient pressure and the periodic resonator motion (deformations are exaggerated).

where $\omega_0^2 = k/m$ and $Q = m\omega_0/b$. The equivalent stiffness $k$ is the sum of a mechanical stiffness $k_{mech}$ and an electrostatic stiffness $k_{elec} < 0$

$$k_{elec} = -\frac{c_{eq}}{d^2}V_b^2 \quad (2)$$

where $d$ is the electrostatic gap, $V_b$ is the bias voltage applied to the resonator and $c_{eq}/d$ is the electromechanical transduction coefficient. Assuming no stress stiffening, we have

$$\gamma = \frac{2}{d^2}\frac{\frac{k_{elec}}{k_{mech}}}{1+\frac{k_{elec}}{k_{mech}}} \quad (3)$$

$$g_{drv} = \frac{c_{eq}}{m}\frac{V_b}{d} \quad (4)$$

This model is valid provided $(x/d)^2 \ll 1$, and thus covers a large range of operation above the critical Duffing amplitude $(x/d)^2 \gtrsim 1/Q$. Close to resonance, one may use harmonic balance to transform (1) in

$$2j\Omega\dot{X} - \Omega^2 X \approx -\left(\omega_0^2\left(1+\frac{3}{4}\gamma|X|^2\right) + j\Omega\frac{\omega_0}{Q}\left(1+\frac{\alpha}{4}|X|^2\right)\right)X + g_{drv}V_{drv} \quad (5)$$

where $X$ (respectively $V_{drv}$) is the slowly-varying complex amplitude of $x$ (respectively $v_{drv}$). Finally, the complex amplitude of the measured voltage is a linear combination of $X$ and $V_{drv}$

$$V_{meas} = g_{sen}X + g_{ft}V_{drv} \quad (6)$$

where $g_{sen}$ and $g_{ft}$ are transduction (sensing) and feedthrough coefficients. Note that because of front-end non-idealities, both $g_{sen}$ and $g_{ft}$ are usually complex-valued. With an ideal charge amplifier with capacitance $c_{sen}$, we have

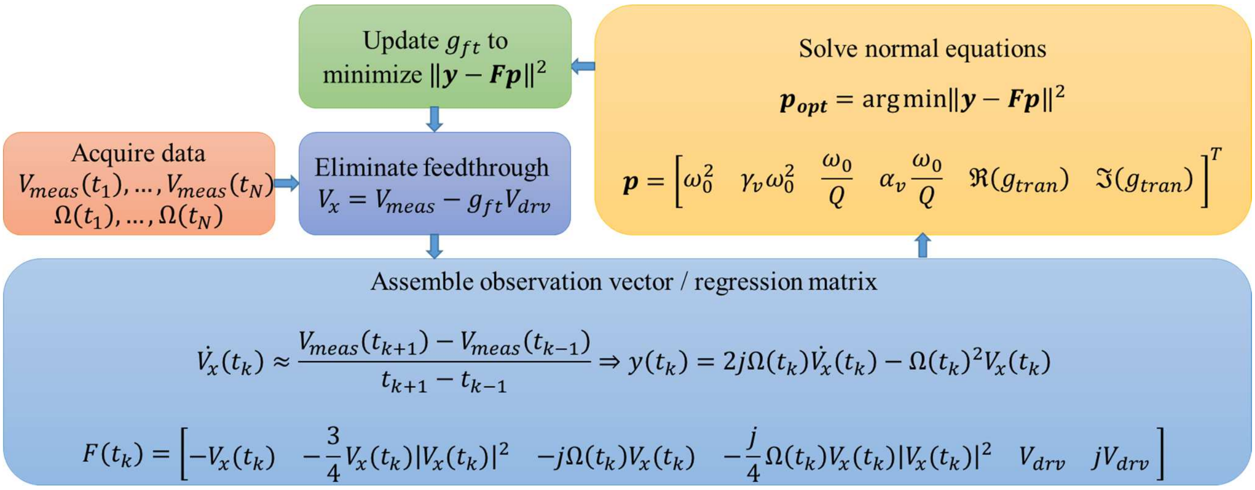


Fig. 2. Parameter estimation procedure for frequency sweep data.

$$g_{sen} = \frac{c_{eq}}{c_{sen}}\frac{V_b}{d} \tag{7}$$

and

$$g_{ft} = \frac{c_{ft}}{c_{sen}} \tag{8}$$

Letting $V_x = V_{meas} - g_{ft}V_{drv}$, one may condense (5) and (6) into

$$2j\Omega\dot{V}_x - \Omega^2 V_x \approx -\left(\omega_0^2\left(1 + \frac{3}{4}\gamma_v|V_x|^2\right) + j\Omega\frac{\omega_0}{Q}\left(1 + \frac{\alpha_v}{4}|V_x|^2\right)\right)V_x + g_{tran}V_{drv} \tag{9}$$

where $g_{tran} = g_{sen}g_{drv}$ , $\alpha_v = \alpha/|g_{sen}|^2$ , and $\gamma_v = \gamma/|g_{sen}|^2$.

## III. Testing procedure

### A. Data processing

The purpose of the testing procedure is to determine as much information as possible on the system parameters within a given amount of time. Note that parameters $g_{tran}$ and $g_{ft}$ are only accessible through the forced excitation of the resonator.

#### 1) Fast frequency sweeps

Here we estimate all four real parameters $\omega_0, Q, \gamma_v$ and $\alpha_v$ in the above model, along with the two complex parameters $g_{tran}$ and $g_{ft}$ by sweeping the driving frequency $\Omega$ across the resonant bandwidth, with a constant drive amplitude $V_{drv}$.

The processing of the measured voltage $V_{meas}$ boils down to:

- estimating $\dot{V}_x$ (which is equal to $\dot{V}_{meas}$ since $\dot{V}_{drv} = 0$) with a finite difference approximation.
- estimating the parameters in the right-hand side of (9) – i.e. $\omega_0^2$ , $\gamma_v\omega_0^2$ , $\omega_0/Q$ , $\alpha_v\omega_0/Q$ , $\Re(g_{tran}), \Im(g_{tran})$ – by solving (9) in a least-squares sense. This is a linear least squares problem, which can be solved analytically, with 6 unknowns.

Note that $\Re(g_{ft})$ and $\Im(g_{ft})$ must be known in order to determine $V_x$ from $V_{meas}$ and solve (9) for the other parameters. In practice, these 2 parameters are determined iteratively along with the other 6. This is achieved thanks to an 8-dimensional least squares procedure, nonlinear in $\Re(g_{ft})$ and $\Im(g_{ft})$, linear in the other 6 parameters, in which a 6-dimensional linear least squares problem is repeatedly solved (Fig.2). Although solving for the feedthrough coefficients is an added difficulty compared to free decay methods, the problem is well-conditioned and we have found the nonlinear least squares procedure always converges very fast, regardless of the initial guess for $g_{ft}$.

#### 2) Free decay response

Here the resonator is driven to steady-state at a fixed frequency $\approx \omega_0$ with a constant drive amplitude $V_{drv}$. The drive is then turned off, and the free decay response is exploited to determine the system parameters (with the obvious exception of $g_{tran}$ and $g_{ft}$ ). The processing is performed in three steps:

- $\dot{V}_x$ is estimated with a finite difference approximation, as above.
- a first estimation of the 4 unknown parameters - $\omega_0^2, \gamma_v\omega_0^2, \omega_0/Q, \alpha_v\omega_0/Q$ - is determined by solving (9) in a least squares sense (as above, a linear least squares problem), to provide an initial guess for the third step.
- the estimates are refined by fitting the analytical solution [8-9] of (9) when $V_{drv} = 0$ and $\dot{\Omega} = 0$ to the measured voltage. This step generally improves the precision of the estimates obtained previously.

The latter step is actually a nonlinear least squares problem with 6 unknowns: 4 system parameters and 2 initial conditions. There is no convergence issue provided a proper initial guess is available, hence the importance of the second processing step.

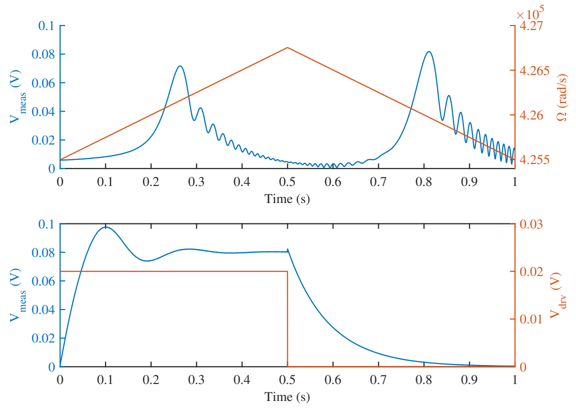


Fig. 3. Typical waveforms (blue) and stimuli (red) for sweeps (top) and ring-downs (bottom).

## B. Experimental setup

The resonator and its readout are operated at ambient temperature and pressure. An arbitrary function generator (Tektronix AFG3052C) is used to drive the resonator either with repeated frequency sweeps or frequency bursts. The amplitude $V_{drv}$ of the quasi-harmonic driving voltage is kept constant at 20mV for both types of excitation. The frequency sweeps last 1s, spanning a 200Hz range ($\approx 114 \times \Delta f$), and are split equally in an upward 500ms ($\approx 5.5 \times \tau$) linear sweep immediately followed by a downward 500ms linear sweep. The frequency bursts consist in 500ms of excitation at a fixed frequency followed by a 500ms ring-down phase during which $V_{drv} = 0$. A mixed-signal oscilloscope (Tektronix MSO54), properly synchronized to the waveform generator, is used to measure the signals output by the resonators, demodulate them with a 5kHz bandwidth and compute their instantaneous amplitude and phase to yield the slowly-varying complex signal $V_{meas}$ (sampled at a rate of 6250Hz). Each stimulus is repeated 100 times, with 1s intervals, for a given value of the DC bias voltage $V_b$, to assess the consistency of the estimates obtained with either approach and their precision.

The main difficulty for this set of experiments is with ring-downs, the estimates of $\alpha_v$ and $\gamma_v$ derived from free decay responses being quite dependent on the initial amplitude of the resonator, as illustrated in section IV. Thus, for each value of $V_b$, the frequency of the drive signal must be precisely adjusted to the resonance frequency of the resonator, which is dependent on $V_b$. On the other hand, sweeps generally yield consistent and precise estimates as long as the resonator's resonance frequency is inside the swept bandwidth.

Typical experimental waveforms are reported in Fig. 3. Qualitatively, the dissimilarity between the upward ($t$<0.5s) and downward ($t$>0.5s) sweep response is a clear indication of the nonlinear (softening) characteristic of the resonator.

## IV. Results

We represent in Fig. 4 the parameters estimated with sweeps and ring-downs for different values of $V_b$, with error bars corresponding to $\pm\sigma$ where $\sigma$ is the 2s-Allan deviation determined from the 100 successive tests at each value of $V_b$.

The estimates of "linear" parameters ($f_0$ or $Q$) obtained with either approach are very consistent with each other. The slight difference in the estimated values of $f_0$ (about 2Hz) or

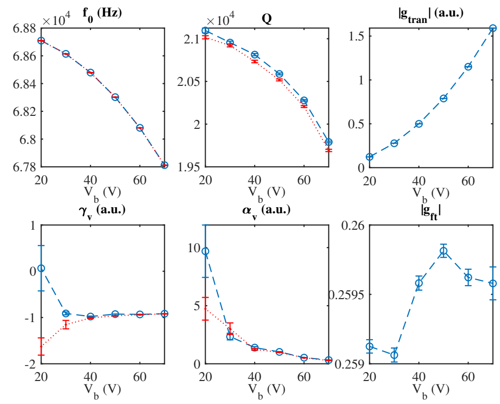


Fig. 4. Mean value of the estimated parameters vs. bias voltage (blue circles : sweeps, red dots : ring-downs). The values of $g_{tran}$, $\gamma_v$ and $\alpha_v$ are normalized.

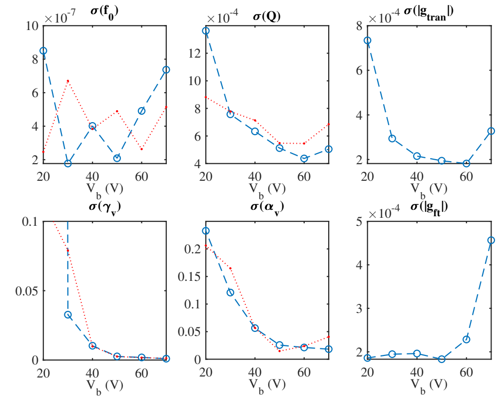


Fig. 5. Precision of the estimated parameters vs. bias voltage (blue circles : sweeps, red dots : ring-downs).

$Q$ (about 100) is coherent with a change of operating temperature between the sweeps and the bursts. Likewise, the $f_0$ vs. $V_b$ and $Q$ vs. $V_b$ relations are coherent with electrostatic softening, as captured in the model of section II.

The estimates of $\gamma_v$ and $\alpha_v$ obtained with either approach are also consistent with each other, provided $V_b \geq 40V$. Below this value, the drive amplitude is not large enough for nonlinearity to have a significant impact on the dynamics of the resonator and, consequently, the estimation of $\gamma_v$ and $\alpha_v$ is rather poor. Note however that this poor estimation of the nonlinear coefficients at low $V_b$ does not seem to have a negative impact on the estimation of the other parameters. The relation between $\gamma_v$ and $V_b$ is also coherent with the model of section II.

Finally the estimates of $|g_{tran}|$ and $|g_{ft}|$ obtained with sweeps are also represented, once again with a behavior coherent with the proposed model.

In Fig. 5, we represent the precision, or relative error, of the estimated parameters, defined as the ratio of the 2s-Allan deviation to the average value of the considered parameter. For all parameters, the precision is roughly the same whether sweeps or ring-downs are used. The precision of $f_0$ (min.

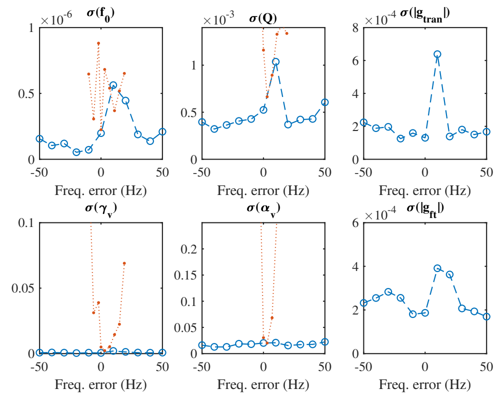


Fig. 6. Precision of estimates vs. frequency error (blue circles : sweeps, red dots : ring-downs).

value 200ppb) is essentially limited by the fluctuations of the (uncontrolled) ambient pressure. The precision of $\gamma_v$ improves monotonously with $V_b$, down to 0.1%, and the same goes for the precision of $\alpha_v$ (down to 2%). On the other hand, there seems to be an optimum between 50V and 60V for $Q$ (400ppm), $|g_{tran}|$ (200ppm) and particularly for $|g_{ft}|$ (200ppm). The fact that precision deteriorates for large values of $V_b$ may be due to a number of, as yet uninvestigated, causes (fluctuations of $V_b$, model validity for large oscillation amplitudes, too large "signal to feedthrough" ratio to properly estimate $g_{ft}$, etc.)

It should be stressed that, while sweep estimates are quite robust to the exact position of the resonance frequency relatively to the swept bandwidth, ring-down estimates, especially of $\alpha_v$ and of $\gamma_v$ are very sensitive to the exact value of the frequency of the drive during the ring-up phase. In this respect, the results in Fig. 4-5 correspond to

- the best-case estimates that can be obtained with nonlinear ring-down, when the driving frequency is carefully adjusted (to within $\pm$0.5Hz of $f_0$) in order to maximize the ring-down response.
- typical estimates that can be obtained with sweeps, the estimated resonance frequency being a posteriori found to be within 50Hz from the central sweep frequency.

To stress this point, we represent in Fig. 6 the influence of "frequency error" – corresponding to the difference between $f_0$ and either (in the case of sweeps) the central sweep frequency or (in the case of ring-downs) the ring-up frequency – on the precision of all parameter estimates, when the driving frequency is intentionally misadjusted. These results are obtained at $V_b$ =70V, from 100 consecutive ring-downs / 20 sweeps at each central/ring-up frequency, and should be compared to the last point of Fig. 5. This experiment demonstrates that estimates derived from frequency sweeps are largely unaffected by frequency error. It also shows that nonlinear ring-down is not a robust technique when it comes to estimating nonlinear parameters – unless specific precautions are taken (e.g. closed-loop driving to resonance during the ring-up phase, as in [8-9]): the precision with which $\gamma_v$ is estimated quickly deteriorates to more than 10%, that of $\alpha_v$ to more than 100%, as also illustrated in Fig. 7. The precision with which $Q$ is estimated by nonlinear ring-down also deteriorates with frequency error, but not as dramatically.

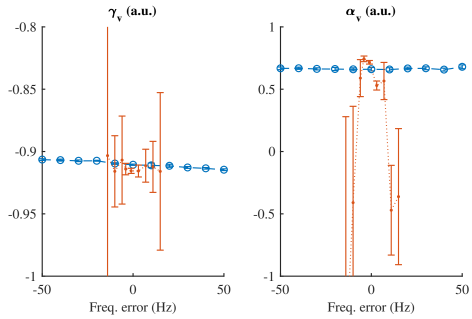


Fig. 7. Mean value of $\gamma_v$ and $\alpha_v$ vs. frequency error (blue circles : sweeps, red dots : ring-downs).

## V. Conclusion

In this paper, we have described and compared two time-efficient techniques, based on a Hilbert transform formalism, for estimating the parameters of nonlinear (MEMS) resonators, according to two scenarios: forced excitation, on the one hand, free decay, on the other hand. We have shown how, in the first scenario, parameter estimation boils down to a well-behaved two-dimensional nonlinear least squares problem (for feedthrough parameters) coupled to an equally well-behaved six-dimensional linear least squares problem.

The two techniques were assessed experimentally and the consistency of the estimated parameters was verified. The precision with which parameters could be estimated was 200ppb for the resonance frequency (limited by environmental fluctuations), 400ppm for the quality factor, 200ppm for both transduction and feedthrough gain (not estimated with ring-down), 0.1% for the Duffing coefficient and 2% for the nonlinear damping coefficient.

Note that the reported measurements were performed over a duration approximately equal to $11 \times \tau$, which was considered "convenient" with respect to a number of factors (total duration of the experiments, available equipment, size of the data files…). We verified that increasing or reducing this duration mostly impacted the precision with which linear and nonlinear damping coefficients were estimated, in proportion with the signal-to-noise ratio, while leaving the precision of the other parameters unchanged. However, this behavior is likely setup- or device-dependent and entails further study.

In the case of ring-down, it was shown that the precision with which nonlinear coefficients are estimated is extremely dependent on the prior knowledge of the resonator's frequency. This highlights that nonlinear ring-down is essentially a closed-loop technique, whose performance is, at least in part, conditional to the way ring-up is performed. On the other hand, the estimates obtained with frequency sweeps are very robust. This shows that fast frequency sweeps may be used as a purely open-loop parameter estimation technique, with little prior knowledge on the resonator parameters. Several applications may benefit from this technique: for example, in a context of industrial test, our approach enables the simultaneous characterization of a large number of resonators in the same environment, with the same stimulus. The precision with which most parameters can be estimated also indicates some perspectives for sensing applications. For example, the most common electronic architecture in resonant strain gauge applications is the self-oscillating loop (whether PLL-based or not): but our open-loop approach provides $f_0$

with a precision that is comparable to that of a self-oscillating loop over the same bandwidth, and, in addition, also provides precise information on the other system parameters (thus making it possible to compensate for influence quantities such as bias voltage or temperature variations). Shifting from a closed-loop architecture to an open-loop one may also alleviate some issues associated to self-oscillating loops (feedback noise, A-f effect, etc.) In the same vein, one may also use fast frequency sweeps to re-visit the concept of open-loop mode-localized sensors [16-17] with a predictable gain in bandwidth.

The hardware implementation of our technique in an embedded system, its extension to resonators with more complex behavior (e.g. accounting for detection and actuation nonlinearity, and for multiple degrees of freedom, as in mode-localized sensors or gyroscopes) and the development of different variants are parts of our ongoing work.

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